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by

Yu-Chung Liao

June 1982 LCDS #82-17

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Abstract

This report discusses

~~Consider~~ a controlled diffusion process which evolves as a reflected Brownian motion under each control action. A switching cost is incurred when the control action is switched. The control problem turns out to be a sequential decision problem, i.e., to find a sequence of optimal stopping times to switch control. The dynamic programming equation for a discounted cost criterion is a quasi-variational inequality. By allowing the discount factors tend to zero, we show a new Q.V.I. has a solution that serves as a potential function to give direction to attain the optimality for a long-run average cost criterion.

Key words: diffusion, switching cost, quasi-variational inequality, potential function, long-run average cost.

1. Introduction

Optimal control of reflected Brownian motion arises naturally from input-output systems. Faddy [4] models a dam by a Brownian motion with two reflecting barriers. Puterman [9] uses diffusion processes to model production and inventory processes. In both cases they assume the existence of a stationary optimal strategy and start from there. In Rath [10] a Bang-Bang style strategy is proved to be optimal among stationary strategies by using a random walk to approximate Brownian motion. Chernoff and Petkau [2] prove that the optimal conditions are satisfied by certain strategies. All those papers discuss the case of linear holding costs and two control actions.

Here, we consider a controlled diffusion process which evolves as a reflected Brownian motion. A switching cost is incurred when the control action is switched. Since the instants of switches are crucial, the optimal control problem turns out to be a sequential decision problem. We can write the dynamic programming equation for a discounted cost criterion by the principle of dynamic programming in Fleming-Rishel [5]. It is a quasi-variational inequality which can be solved by the penalty method in Bensoussan-Lions [1]. By allowing the discount factors tend to zero, a new quasi-variational inequality arises as the dynamic programming equation for a long-run average cost criterion. We solve it to prove the existence of a stationary optimal strategy.

2. Model

Let (Ω,F,P) be a probability space on which a standard Brownian motion W_t is defined. $\{F_t\}_{t=0}^{\infty}$ is the increasing family of complete σ-fields generated by W_t. Let S be the set of F_t-stopping times. Let $A = \{1,2,\ldots M\}$ be the set of control actions. Under control action i, the controlled process evolves as the reflected Brownian motion

$$(1) \qquad Rf(x+d_i t+a_i W_t)$$

where Rf is a function on $C[0,\infty)$ defined as

$$Rf(w)(t) = w(t) - \inf\{0; w(s), s \le t\}$$

for all $w \in C[0,\infty)$. The operating and holding cost is $f(x,i)$ per unit time if the state of the process is x and action i is used. When switching from action i to j a switching cost $C(i,j)$ is incurred. Since infinitely many switches in a finite time interval will make the total cost blow up, we can, without loss of generality, define a strategy $u = \{s(n),u(n)\}_{n=1}^{\infty}$ to be admissible if

i) $s(n) \in S$ for all n,

ii) $0 \le s(n) < s(n+1)$ for all n

iii) $s(n) \to \infty$ as $n \to \infty$ w.p.1.,

iv) $u(n):\Omega \to A$ is $F_{s(n)}$-measurable and $u(n) \neq u(n+1)$ for all n.

Then given initial state (x,i) and admissible strategy u the control process is

$$u(t) = \begin{cases} i & t < s(1), \\ u(n) & s(n) \le t < s(n+1) \end{cases}$$

and the controlled process is

$$Rf(x+\int_0^t d_{u(s)}ds+\int_0^t a_{u(s)}dW_s).$$

We assume the following conditions throughout this paper.

(2) $a_i \neq 0 \quad i \in A,$

(3) $d_i < 0 \quad i \in A,$

(4) $f(\cdot,i):R^+ \to R^+$ is bounded measurable and nondecreasing $i \in A$,

(5) $C(i,j) > 0$ and $C(i,j) + C(j,l) > C(i,l) \quad i \neq j$ and $j \neq l$.

Here, (3) is a stability condition. See Kushner [7] and [8].

3. Preliminary Results.

To use variational inequality techniques for solving sequential decision problems has been studied extensively in [1]. For completeness we briefly discuss some results in a form which is suited for use in the next section.

Let G be an open subset of R^+, $\mu \geq 0$, $p \geq 1$ and $D = \frac{d}{dx}$. We denote the space of all functions f on G such that

$$\sum_{k=0}^{n} \|e^{-\mu x}D^k f(x)\|_{L^p(G)} < \infty$$

by $W^{n,p,\mu}(G)$ and $W^{n,p}(G)$ if $\mu = 0$. Since the generalized Itô's formula in [1] holds for the diffusion processes with reflecting boundary in Stroock-Varadhan [13], the following theorem is proved by the penalty method in [1]. We state it without proof.

Let I_Q be the indicator function of the set Q, $\|\cdot\|_G$ be the sup-norm in $L^\infty(G)$ and

$$L^i = \frac{-1}{2} a_i^2 D^2 - d_i D \qquad i \in A.$$

<u>Theorem 1.</u> <u>Let</u> $G = [0,B]$, $r > 0$, $f \in L^\infty(G)$, $v \in R$ <u>and</u> h <u>satisfy</u>

(6)
(a) $h \in W^{2,\infty}(G)$,
(b) $Dh(0) = 0$,
(c) $v \leq h(B)$.

<u>Then the variational inequality</u>

(7)
(a) $Z \in W^{2,p}(G) \quad 1<p<\infty$,
(b) $L^i Z + rZ - f \leq 0$ a.e. <u>on</u> G,
(c) $Z - h \leq 0$,
(d) $(b)\times(c) = 0$,
(e) $DZ(0) = 0$
(f) $Z(B) = v$

<u>has a unique solution for any fixed</u> $i \in A$. <u>In addition we have</u>

(8) $$\|L^i Z\|_G \leq |v| + 2\|f\|_G - d_i\|Dh\|_G + \frac{1}{2} a_i^2 \|(D^2h)^+\|_G$$

<u>and</u>

(9) $$Z(x) = \inf_{s \in S} E_x\{\int_0^{s\wedge\tau} e^{-rt} f(x(t))dt + e^{-rs} I_{\{s<\tau\}} h(x(s)) + e^{-r\tau} I_{\{s\geq\tau\}} v\}.$$

Here, $x(t)$ is the process in (1), and $\tau = \inf\{t : x(t) = B\}$.

Corollary 1. Given $h \in W^{1,\infty}(G)$ and satisfy

(10) there is a constant c and a sequence h_n such that h_n satisfies (6), $D^2 h_n < c$ for all n and $h_n \to h$ as $n \to \infty$ in $W^{1,\infty}(G)$.

Then Theorem 1 holds. In this case, $\|(D^2h)^+\|_G$ in (8) is replaced by $C' = \mathrm{Max}\{0,c\}$.

Proof. Let Z_n be the solution of (7) with respect to h_n. By (8) there is a subsequence of n, still denoted by n, such that $Z_n \to Z$ weakly in $W^{2,p}(G)$ and strongly in $W^{1,p}(G)$. Hence Z satisfies (7), (8) and (9).

We have the same conclusions as in Corollary 1 when the Neumann boundary condition (7e) is replaced by the Dirichlet boundary condition.

Corollary 2. Let $G = [B_1, B_2]$ and $B_1 > 0$. Then Corollary 1 is true if (7e) and (7f) are replaced by

$$(e')\quad Z(B_i) = v_i \text{ and } v_i \leq h(B_i) \quad i = 1,2.$$

In case $r|v_i| \leq \|f\|_G$ for $i = 1,2$ then $|v|$ in (8) can be removed, i.e.

$$\|L^i Z\| \leq 2\|f\|_G - d_i\|Dh\|_G + \tfrac{1}{2}a_i^2 C'. \tag{11}$$

Another result from Robin [11] is

Theorem 2. Given $f \in L^\infty(R^+)$, $r > 0$, h bounded continuous on R^+. Let $x(t)$

be the process in (1) and

$$Z(x) = \inf_{s\in S} E_x\left\{\int_0^s e^{-rt} f(x(t))dt + e^{-rs}h(x(s))\right\}. \tag{12}$$

Then Z is bounded continuous on R^+, $Z \leq h$ and

$$Z(x) = E_x\int_0^\tau e^{-rt} f(x(t))dt + e^{-r\tau}h(x(\tau))$$

where

$$\tau = \inf\{t:Z(x(t)) = h(x(t))\}.$$

The next lemma gives useful estimates.

Lemma 1. *If both f and h in Theorem 2 are nondecreasing and non-negative then so is Z.*

Proof. Given $x > y$ then

$$Rf(x+d_i t+a_i W_t) \geq Rf(y+d_i t+a_i W_t) \quad \text{w.p.1.}$$

By (12), it is clear that $Z(x) \geq Z(y) \geq 0$.

Following the assumptions of Lemma 1 we have

$$0 \leq Z(x) - Z(y) \leq E_x\left\{\int_0^\tau e^{-rt} f(x(t))dt + e^{-\tau t}Z(y)\right\} - Z(y)$$

where

$$\tau = \inf\{t:x(t) = y\}.$$

By Karlin-Taylor [6],

$$E_x \tau = - \frac{1}{d_i}(x-y).$$

Hence

$$0 \leq Z(x) - Z(y) \leq - \frac{1}{d_i} (x-y) \cdot \|f\|_{R^+}$$

and

$$0 \leq DZ(x) \leq - \frac{1}{d_i} \| f \|_{R^+}. \tag{13}$$

Let

$$V_0^r(x,i) = E_x \int_0^\infty e^{-rt} f(x(t),i)dt$$

and

$$V_m^r(x,i) = \inf_{s \in S} E_x \left\{ \int_0^s e^{-rt} f(x(t),i)dt + e^{-rs} M_i V_{m-1}^r(x(s)) \right\} \quad m \geq 1$$

where

$$M_i V_m^r(x) = \min_{j \neq i} C(i,j) + V_m^r(x,j) \qquad m \geq 1.$$

By induction and Lemma 1, $V_m^r(x,i)$ is non-decreasing and non-negative for all i and m. Hence

$$0 \leq DV_m^r(x,i) \leq - \frac{1}{d_i} \|f(x,i)\|_{R^+}. \tag{14}$$

Now $M_i V_m^r(x)$ is satisfied by (10) when $V_m^r(x,j) \in W^{2,\infty}(R^+)$ for all $j \in A$.

By induction, we can choose boundary condition $v = V_m^r(B)$ and use Corollary 1 to show locally, hence globally, that

$$
(15)\quad
\begin{aligned}
&\text{(a)}\quad V_m^r(x,i) \in W^{2,\infty}(R^+) \qquad i \in A \text{ and } m \geq 0,\\
&\text{(b)}\quad V_m^r(x,i) - M_i V_{m-1}^r(x) \leq 0 \qquad i \in A \text{ and } m \geq 1,\\
&\text{(c)}\quad L^i V_m^r(x,i) + rV_m^r(x,i) - f(x,i) \leq 0 \text{ a.e. on } R^+ \quad i \in A \text{ and } m \geq 0,\\
&\text{(d)}\quad (b)\times(c) = 0,\\
&\text{(e)}\quad DV_m^r(0,i) = 0 \qquad i \in A.
\end{aligned}
$$

Here (a) comes from (11) and (14) because the upper bound in (11) is actually independent of G. Let

$$
V^r(x,i) = \inf_{u \in U} E^u_{x,i}\left\{\int_0^\infty e^{-rt} f(x(t),u(t))dt + \sum_{n=1}^{\infty} e^{-rs(n)} C(u(n-1),u(n))\right\}
$$

where U is the set of all admissible strategies and $C(u(0), u(1)) = C(i,u(1))$.

<u>Theorem 3.</u> $V^r(x,i)$ <u>is the unique solution of the following quasi-variational inequality</u>

$$
(16)\quad
\begin{aligned}
&\text{(a)}\quad V^r(x,i) \in W^{2,\infty}(R^+) \qquad i \in A,\\
&\text{(b)}\quad L^i V^r(x,i) + rV^r(x,i) - f(x,i) \leq 0 \quad \text{a.e.} \quad i \in A\\
&\text{(c)}\quad V^r(x,i) - M_i V^r(x) \leq 0 \qquad i \in A,\\
&\text{(d)}\quad (b)\times(c) = 0,\\
&\text{(e)}\quad DV(0,i) = 0 \qquad i \in A.
\end{aligned}
$$

Also $V^r(x,i)$ *is non-decreasing for all* i *and there is a constant* K *independent of* r *such that*

(17) $$\|DV^r(x,i)\|_{R^+} \leq K$$

and

(18) $$\|D^2V^r(x,i)\|_{R^+} \leq K \quad \textit{for all} \quad i \in A.$$

Proof. The same as in Evans-Freidman [3]: $V_m^r(x,i)$ is the optimal cost function to control the process with no more than m switches,

(19) $$\|rV_m^r(x,i)\|_{R^+} \leq \|f(x,i)\|_{R^+} \qquad i \in A$$

and

(20) $$V_m^r(x,i) \to V^r(x,i) \quad \text{as} \quad m \to \infty$$

uniformly on R^+. Now all we need is to estimate $D^2V_m^r(y,i)$ for any $y \in R^+$, $i \in A$ and $m \geq 2$. If

$$V_m^r(y,i) < M_iV_{m-1}^r(y),$$

then there is a neighborhood G of y such that

(21) $$L^iV_m^r(x,i) + rV_m^r(x,i) - f(x,i) = 0 \quad \text{a.e. on } G.$$

By (13) and (19), we have

(22) $$\|D^2V_m^r(x,i)\|_G \leq 6a_i^{-2}\|f(x,i)\|_{R^+}.$$

In case

$$V_m^r(y,i) = M_i V_{m-1}^r(y),$$

there is a set $A' \subset A$ such that $i \notin A'$,

$$V_m^r(y,i) = C(i,j) + V_{m-1}^r(y,j) \qquad j \in A'$$

and

$$V_m^r(y,i) < C(i,j) + V_{m-1}^r(y,j) \qquad j \notin A' \quad \text{and} \quad i \neq j.$$

By (5), we have

$$V_{m-1}^r(y,j) < M_j V_{m-2}^r(y) \qquad j \in A'.$$

So there is a neighborhood G of y on which $V_{m-1}^r(x,j)$ satisfies (21), hence (22), for all $j \in A'$. Thus,

$$(23) \qquad V_m^r(x,i) = \inf_{s \in S} E_x\left\{\int_0^{s\wedge\tau} e^{-rt} f(x(t))dt + e^{-rs} I_{\{\tau>s\}} h(x(s)) + e^{-r\tau} I_{\{\tau\leq s\}} V_m^r(x(\tau),i)\right\} \quad \text{on } G$$

where

$$h(x) = \min_{j \in A'} C(i,j) + V_{m-1}^r(x,j) \quad \text{on } G$$

and

$$\tau = \inf\{t : x(t) \in \partial G \text{ and } x(t) > 0\}.$$

By Corollary 2, (11) and (22), we have

$$\|L^i V_m^r(x,i)\|_G \le 6\,\|f(x,i)\|_{R^+}. \qquad (1.24)$$

From (14), (22) and (24) there is a constant K independent of r and m such that

$$\|DV_m^r(x,i)\|_{R^+} \le K$$

and

$$\|D^2V_m^r(x,i)\|_{R^+} \le K.$$

So the theorem is proved by allowing $m \to \infty$ in (15).

4. Minimum Average Cost Problem.

The total cost to control the process by strategy u up to time T with initial state (x,i) is

$$J(u,x,i,T) = E_{x,i}^u \int_0^T f(x(t),u(t))dt + \sum_{n=1}^{\infty} I_{\{s(n)\le T\}} C(u(n-1),u(n)).$$

The long run average cost is

$$\theta(u,x,i) = \liminf_{T\to\infty} \frac{J(u,x,i,T)}{T}. \qquad (25)$$

The related dynamic programming equation to minimize $\theta(u,x,i)$ is solved as Theorem 4. Theorem 5 is a verification theorem that shows that there is a stationary optimal strategy such that the minimum average cost can be attained as a real limit in (25).

By (19) and (20), there are $j \in A$, $\theta \in R$ and a subsequence of r, still denoted by r, such that

$$V^r(0,j) \le V^r(0,i) \qquad i \in A$$

and

$$rV^r(0,j) \to \theta \quad \text{as} \quad r \to 0.$$

By Theorem 3, we have

$$\begin{aligned} 0 \le \bar{V}^r(x,i) &\equiv V^r(x,i) - V^r(0,j) \\ &\le C(i,j) + V^r(x,j) - V^r(0,j) \\ &\le C(i,j) + Kx. \end{aligned}$$

Since $\bar{V}^r(x,i)$ has the same derivatives as $V^r(x,i)$ has, we have

$$\text{(26)} \quad \begin{aligned} &\text{(a)} \quad \bar{V}^r(x,i) \in W^{2,\infty}(R^+) \quad i \in A, \\ &\text{(b)} \quad L^i\bar{V}^r(x,i) + rV^r(x,i) - f(x,i) \le 0 \quad \text{a.e. on } R^+ \quad i \in A, \\ &\text{(c)} \quad \bar{V}^r(x,i) - M_i\bar{V}^r(x) \le 0 \quad i \in A, \\ &\text{(d)} \quad \text{(b)}\times\text{(c)} = 0, \\ &\text{(e)} \quad D\bar{V}^r(0,i) = 0 \quad i \in A. \end{aligned}$$

By (17), there is a function $V(x,i)$ and a subsequence of r, still denoted by r, such that

$$\bar{V}^r(x,i) \to V(x,i)$$

and

$$rV^r(x,i) \to \theta$$

uniformly on compact subsets of R^+ as $r \to 0$ for all i.

Theorem 4. $V(x,i)$ satisfies

(a) $V(x,i) \in W^{2,p,\mu}(R^+)$ $p > 1$, $\mu > 0$,

(b) $L^i V(x,i) + \theta - f(x,i) \leq 0$ a.e. on R^+ $i \in A$,

(c) $V(x,i) - M_i V(x) \leq 0$,

(d) $(b)\times(c) = 0$,

(e) $DV(0,i) = 0$ $i \in A$

and

$$0 \leq V(x,i) \leq C(i,j) + Kx. \tag{27}$$

Proof. Let $r \to 0$ in (26).

Let g be a twice continuous differentiable function on R^+ such that

$$g(x) \geq 0 \quad \text{on} \quad R^+,$$

$$Dg(0) = 0,$$

$$g(x) = e^{\alpha x} \quad x \geq B$$

and

$$g(x) + |Dg(x)| + |D^2 g(x)| < K' \quad x \leq B$$

for some constants B and K'.

Lemma 1.2. For any u,x and i, $E^u_{x,i} x(T)$ is a bounded function of T.

Proof. By (3), there is an $\alpha > 0$ and $\beta < 0$ such that

$$\frac{1}{2} a_i^2 \alpha^2 + d_i \alpha < \beta \quad i \in A.$$

Let $u = \{s(n),u(n)\}_{n=1}^{\infty}$ with $s(m) = \infty$ for some $m > 0$. By Stroock-Varadhan [21],

$$h(T) \equiv E^u_{x,i} g(x(T))$$
$$= g(x) + E^u_{x,i}\left\{\sum_{n=1}^{m} \int_{T\wedge s(n-1)}^{T\wedge s(n)} -L^{u(t)} g(x(t))dt\right\}$$
$$= g(x) + E^u_{x,i} \int_0^T -L^{u(t)} g(x(t))dt$$

is finite for all T. So

$$dh(T) = E^u_{x,i} - L^{u(T)} g(x(T)$$
$$= E^u_{x,i}\left\{I_{\{x(T)>B\}}\left(\frac{1}{2}\ a^2_{u(T)}\alpha^2 + d_{u(T)}\alpha\right)e^{\alpha x(T)}\right.$$
$$\left. + I_{\{x(T)\leq B\}} - L^{u(T)} g(x(t))\right\}.$$

Hence $Dh(T) < 0$ if

$$h(T) > K'' > K' - \frac{1}{\beta} K'\left(\frac{1}{2}\ a_i^2 - d_i\right) \quad i \in A$$

for some constant K''. Thus

$$h(T) \leq K'' + g(x) \quad T \geq 0.$$

For any admissible strategy u, we have

$$E^u_{x,i} g(x(T)) = \lim_{m\to\infty} E^u_{x,i} I_{\{s(m)\leq T\}} g(x(T))$$
$$\leq K'' + g(x).$$

This proves the lemma.

Let $u^* = \{s(n),u(n)\}_{n=1}^{\infty}$ be defined by

$$s(1) = \inf\{t \geq 0 : V(x(t),i) = M_i V(x(t))\},$$

$$u(1) = \min\{j \in A : V(x(s(1)),i) = C(i,j) + V(x(s(1)),j)\}$$

and

$$s(n) = \inf\{t > s(n-1) : V(x(t),u(n-1)) = M_{u(n-1)} V(x(t))\},$$

$$u(n) = \min\{j \in A : V(x(s(n)),u(n-1)) = C(u(n-1),j) + V(x(s(n)),j)\}$$

for $n > 1$.

<u>Theorem 5.</u> $\theta(u^*,x,i) = \theta \leq \theta(u,x,i)$ <u>for any admissible strategy</u> u.

<u>Proof.</u> By Theorem 4,

$$E_{x,i}^{u^*}\{V(x(T),u(T)) - V(x,i)\}$$

$$(27) \quad = E_{x,i}^{u^*} \sum_{n=1}^{\infty} \{V(x(T\ s(n)),u(T\wedge s(n)) - V(x(T\wedge s(n-1)),u(T\wedge s(n-1)))\}$$

$$= E_{x,i}^{u^*} \sum_{n=1}^{\infty} \left\{\int_{T\wedge s(n-1)}^{T\wedge s(n)} L^{u(n-1)} V(x(t),u(n-1))dt - I_{\{s(n)\leq T\}} C(u(n-1),u(n))\right\},$$

where $u(T\wedge s(0)) = i$. Hence

$$(28) \qquad \theta = \frac{J(u^*,x,i,T)}{T} - \frac{\{V(x,i) - E_{x,i}^{u^*} V(x(T),u(T))\}}{T}$$

and then

$$(29) \qquad \theta = \theta(u^*,x,i)$$

by Lemma 2. To prove $\theta \leq \theta(u,x,i)$ for any u, we simply have inequality at (27), (28) and (29).

<u>Remark.</u> u^* is a stationary strategy.

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REFERENCES

[1] A. Bensoussan and J. Lions, Applications des Inequations Variationelles au Controle Stochastique, Dunod Paris, 1978.

[2] H. Chernoff and A. Petkau, Optimal control a Brownian motion, SIAM J. of Appl. Math., 34(1978), 717-731.

[3] L. Evans and A. Friedman, Optimal stochastic switching and Dirichlet problem for the Bellman equation. Trans. AMS, 253(1979), 365-389.

[4] M. Faddy, Optimal control of finite dams, J. of Appl. Prob., II (1974), 689-710.

[5] W. Fleming and R. Rishel, Deterministic and Stochastic Optimal Control, Springer-Verlag 1975.

[6] S. Karlin and H. Taylor, A First Course in Stochastic Processes, Academic Press, 1974.

[7] H. Kushner, Stochastic Stability and Control, Academic Press, 1967.

[8] H. Kushner, Optimality conditions for the average cost per unit time problem with a diffusion model, SIAM Control 16(1978), 330-346.

[9] M. Puterman, A diffusion model for a storage system, Logistic, M. Geisler ed., North-Holland 1975.

[10] J. Rath, The optimal policy for a controlled Brownian motion process, SIAM J. of Appl. Math., 32(1977), 115-125.

[11] M. Robin, Impulsive Control of Markov Processes, Ph.D. thesis, Paris U. IX, 1978.

[12] D. Stroock and S. Varadhan, Diffusion processes with boundary conditions, Comm. Pure Appl. Math., 24(1971), 147-225.

[13] D. Stroock and S. VAradhan, Multidimensional Diffusion Processes, Springer-Verlag 1979.

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Diffusion; switching cost; quasi-variational inequality; potential function; long-run average cost.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Consider a controlled diffusion process which evolves as a reflected Brownian motion under each control action. A switching cost is incurred when the control action is switched. The control problem turns out to be a sequential decision problem, i.e., to find a sequence of optimal stopping times to switch control. The dynamic programming equation for a discounted cost criterion is a quasi-variational inequality. By allowing the discount factors tend to zero, the author shows a new W.V.I. has a solution that serves as a potential function to give direction to attain the optimality for a long-run average cost criterion.

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